Magnetic Fields
Chapter 29

Magnetic Force on a Current

Current Loops in Magnetic Fields

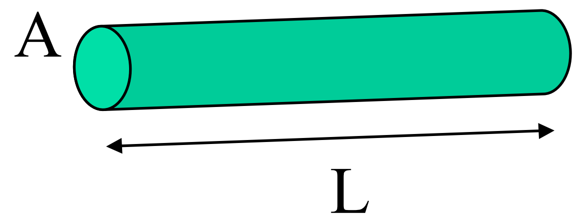
Magnetic Force on a Current

- Force on one charge

$$\underline{F} = q\, \underline{v_d} \times \underline{B}$$

- Forces on all charges

$$\underline{F} = n\, A\, L\, q\, \underline{v_d} \times \underline{B}$$

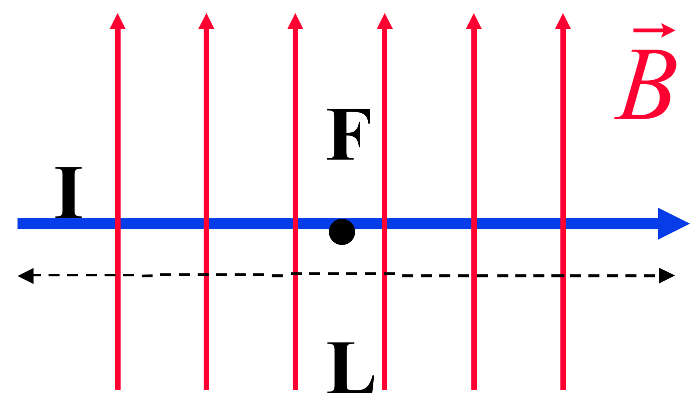


$$I = n\, q\, v_d\, A$$

- But $I = n\, q\, v_d\, A$

- Then $\underline{\mathbf{F}} = \underline{\mathbf{L}}\, \mathbf{I} \times \underline{\mathbf{B}}$

We define a vector $\underline{L}$ whose length is L, and has the same direction as the current I.



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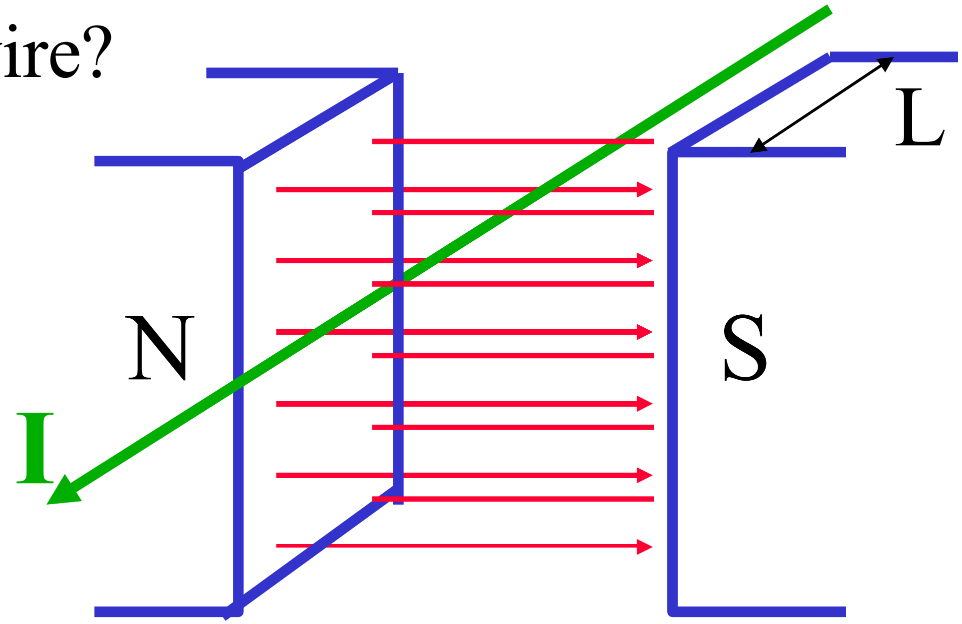
Magnetic Force on a Current

Example: A current, I=10 A, flows through a wire, of length L=20 cm, between the poles of a 1000 Gauss magnet. The wire is at $\theta = 90^0$ to the field as shown. What is the *force* on the wire?



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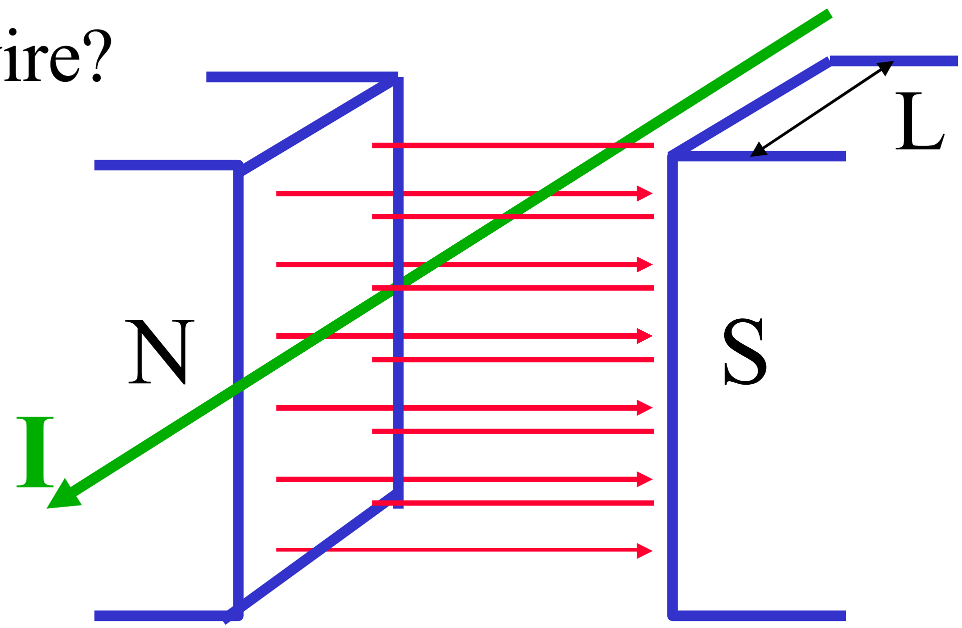
$$\vec{F} = I\,\vec{L} \times \vec{B}$$

$$= ILB \quad \text{(up)}$$



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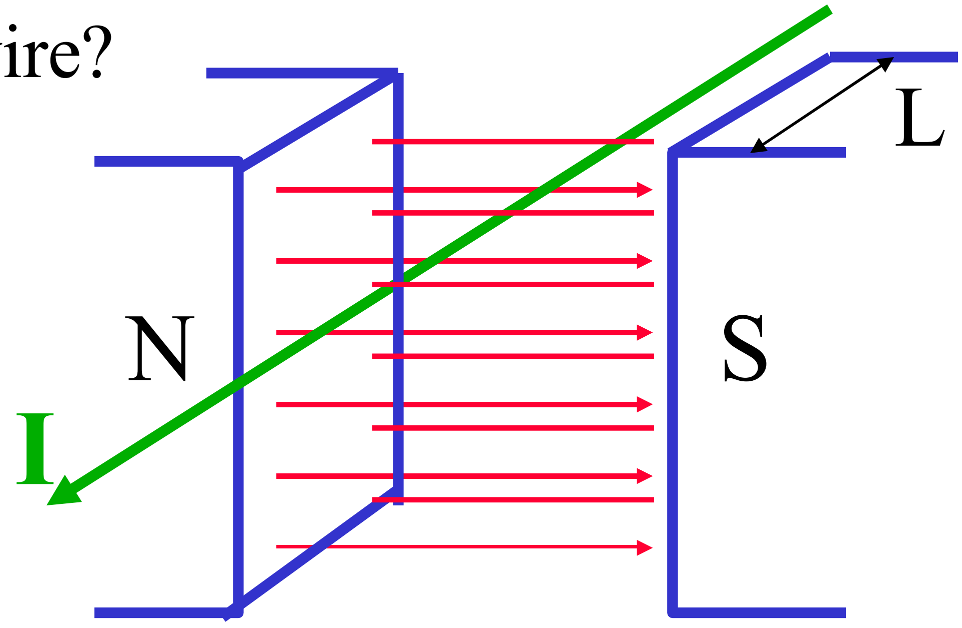
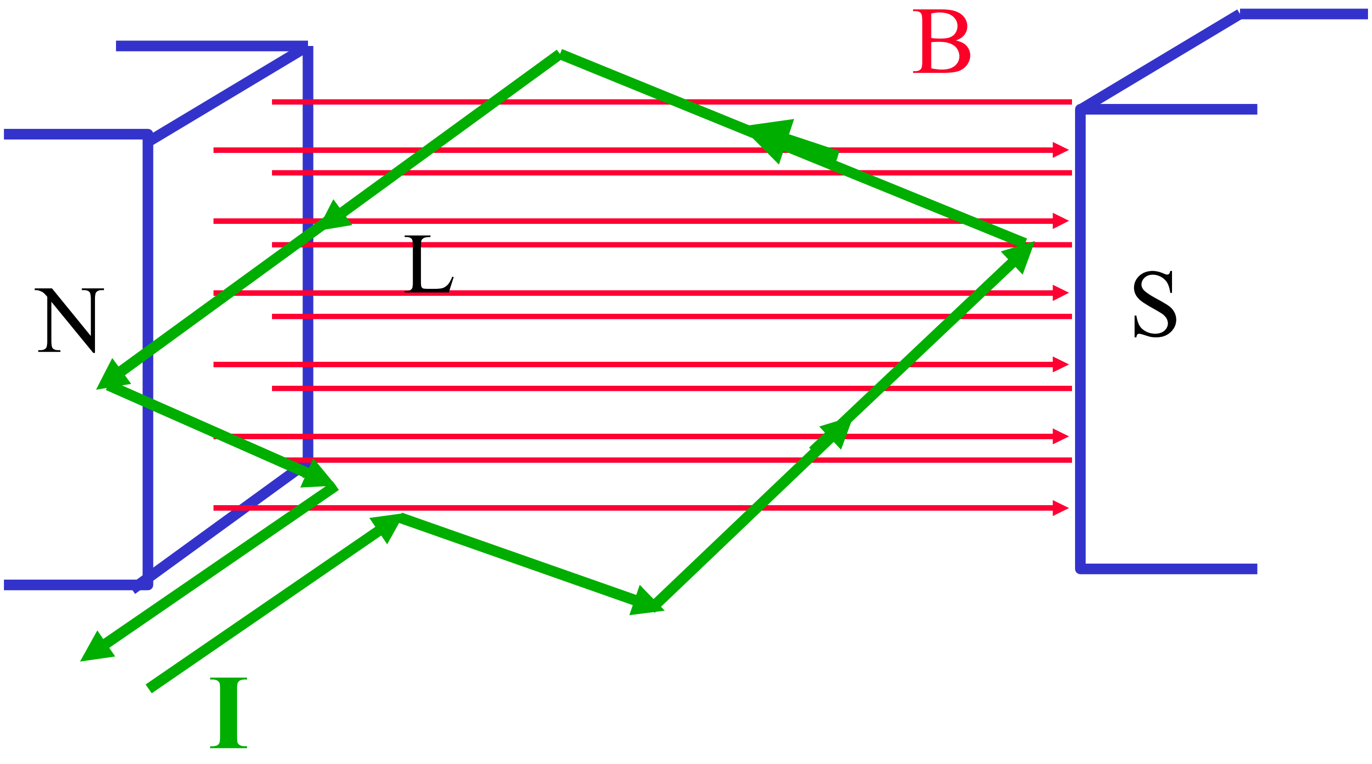


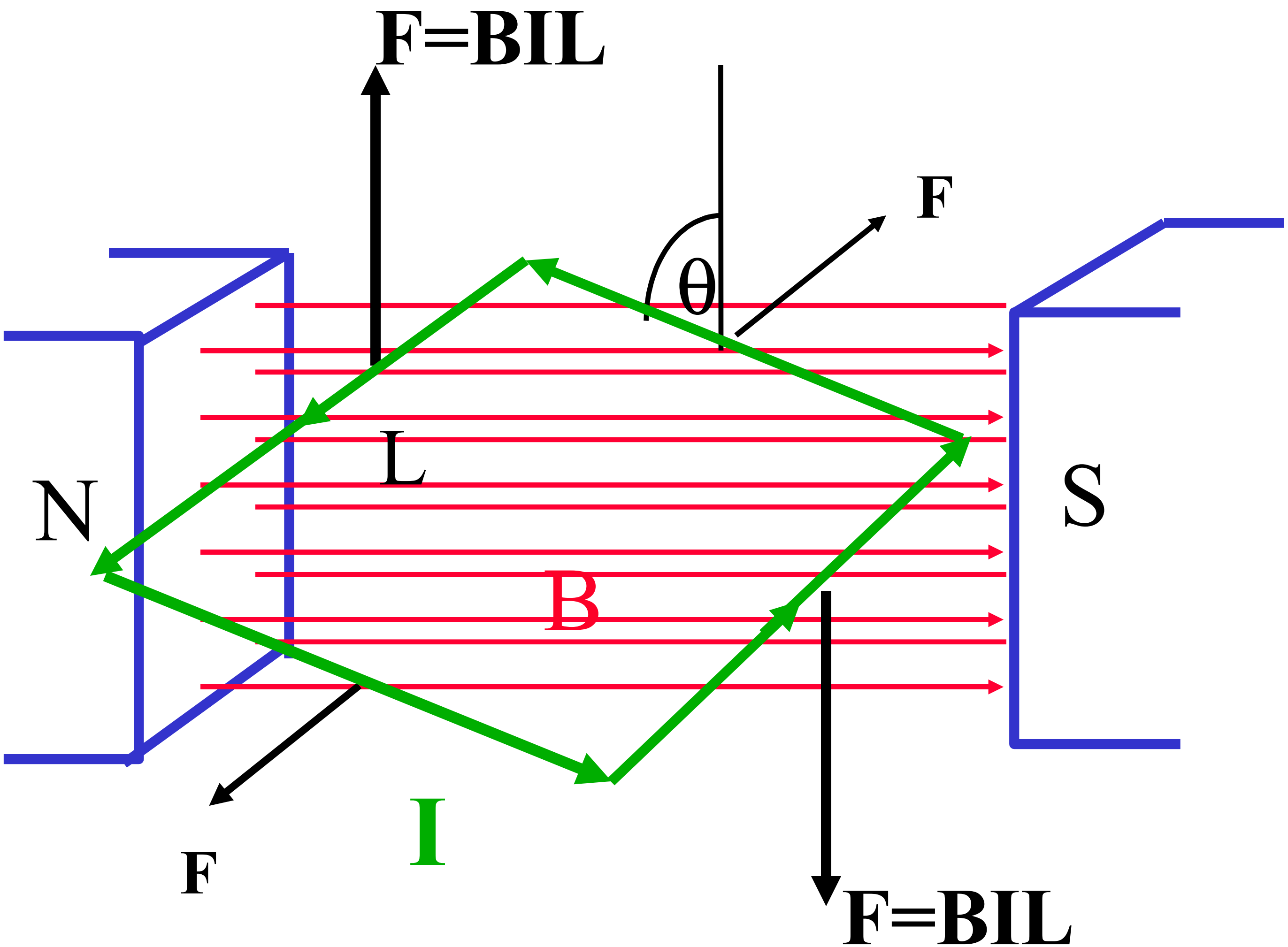
$$= 10(A) \cdot 0.20(m) \cdot 0.1(T) = \underline{\underline{0.2N}} \quad \text{(up)}$$

Magnetic Force on a Current Loop

A current loop is placed in a uniform magnetic field as shown below. What will happen?.

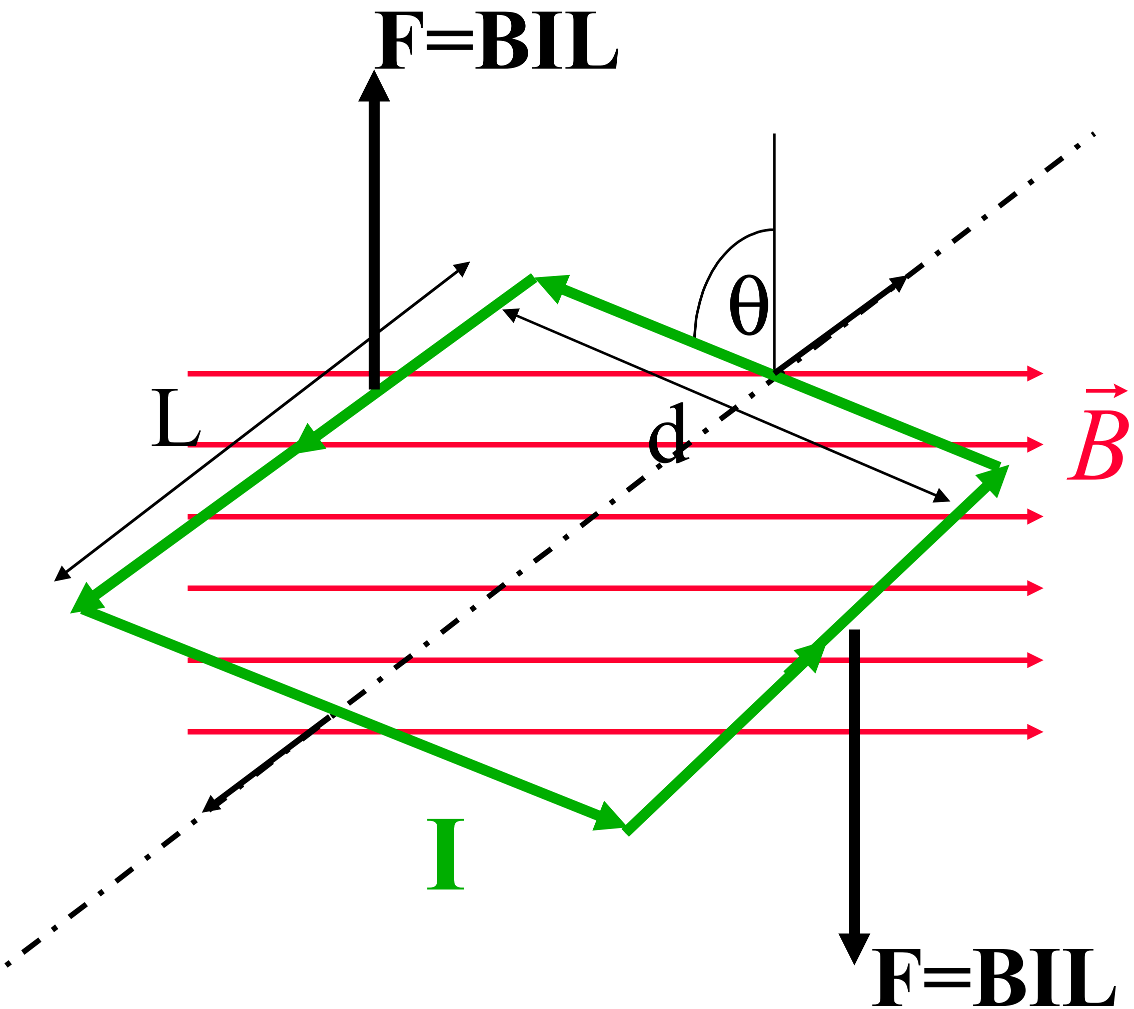


Magnetic Force on a Current Loop



Magnetic Force on a Current Loop

Simplified view:

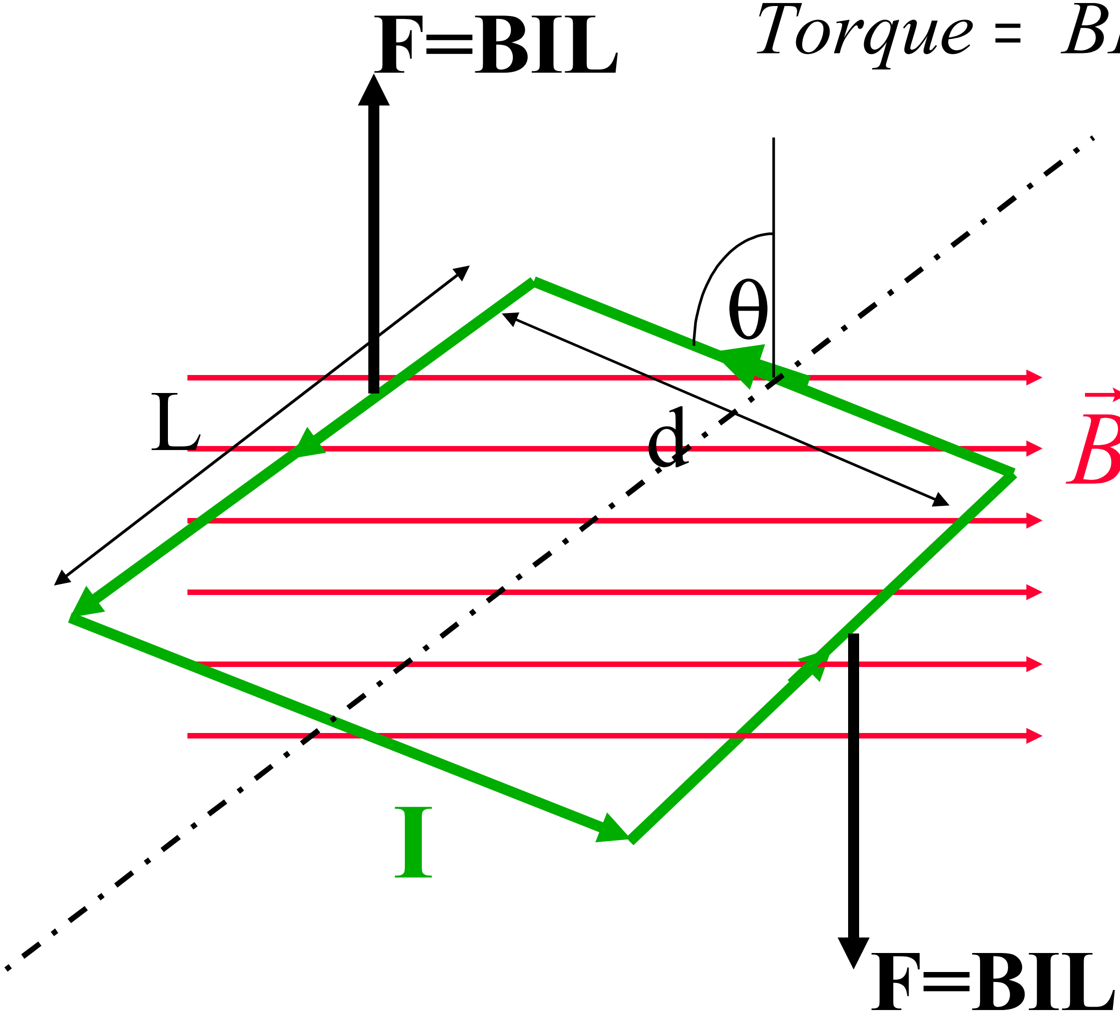


Magnetic Force on a Current Loop
Torque & Electric Motor

Simplified view:

$$Torque = BIL \cdot \left(\frac{d}{2} \sin\theta \right) \cdot 2$$

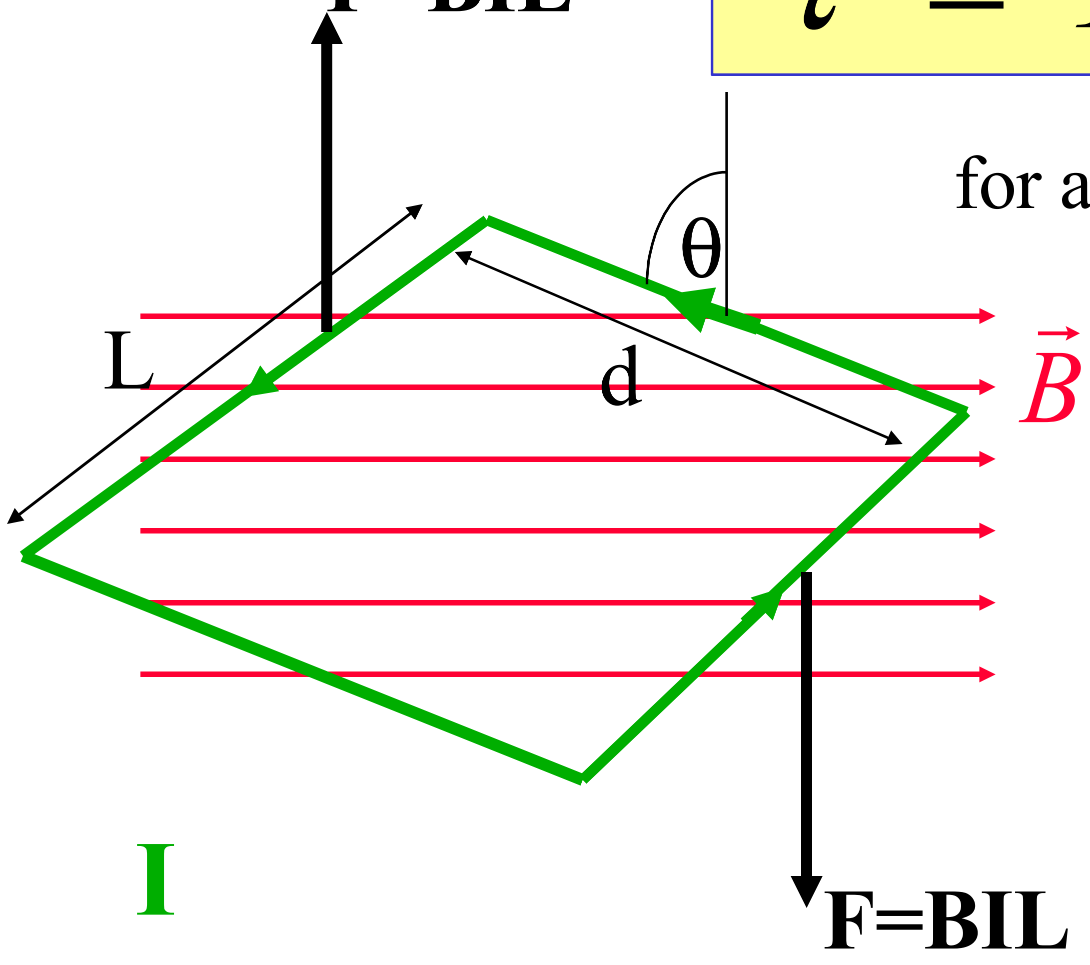
$$= IAB \sin\theta$$



Magnetic Force on a Current Loop
Torque & Electric Motor

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

for a current loop



Magnetic Force on a Current Loop Torque & Magnetic Dipole

By analogy with electric dipoles, for which:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The expression,

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

implies the a current loop acts as a ***magnetic dipole!*** $\vec{\mu} = I\vec{A}$

Here is the magnetic dipole moment,

and

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

(Torque on a current loop)

Potential Energy of a Magnetic Dipole

By further analogy with electric dipoles:

$$U_E = -\vec{p} \bullet \vec{E}$$

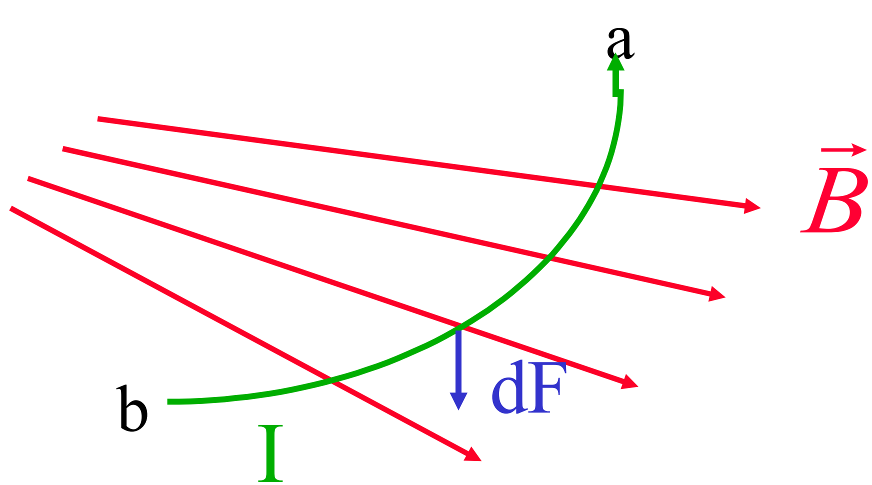
So for a magnetic dipole, (current loop)

$$U_B = -\vec{\mu} \bullet \vec{B}$$

The potential energy is due to the fact that the magnetic field tends to align the current loop perpendicular to the field.

Nonuniform Fields and Curved Conductors

- So far, we have considered only uniform fields and straight current paths.
- If this is not the case, we must again take advantage of calculus;

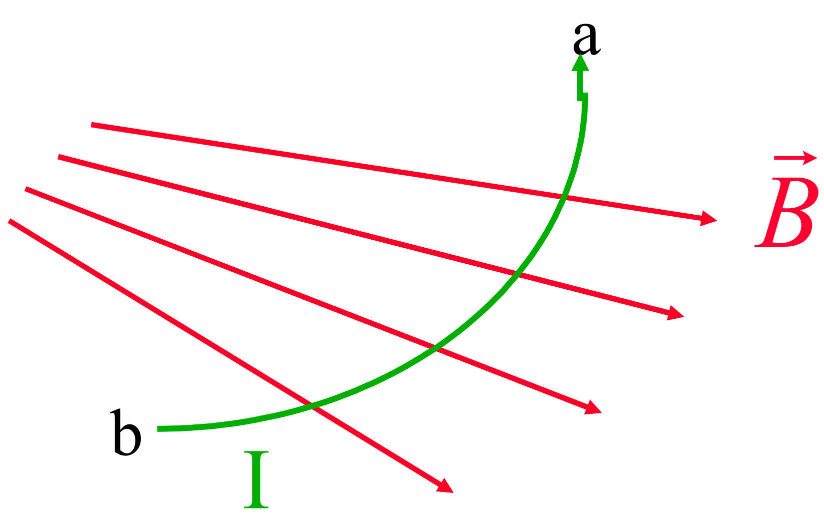


Consider a small length, dL, of current path.

The force on dL is:

$$\underline{dF} = \underline{dL}\ I \times \underline{B}$$

Nonuniform Fields and Curved Conductors



For a conductor of length L

$$\underline{F} = \underline{L}\ I \times \underline{B}$$

For a conductor of length dL

$$\underline{dF} = \underline{dL}\ I \times \underline{B}$$

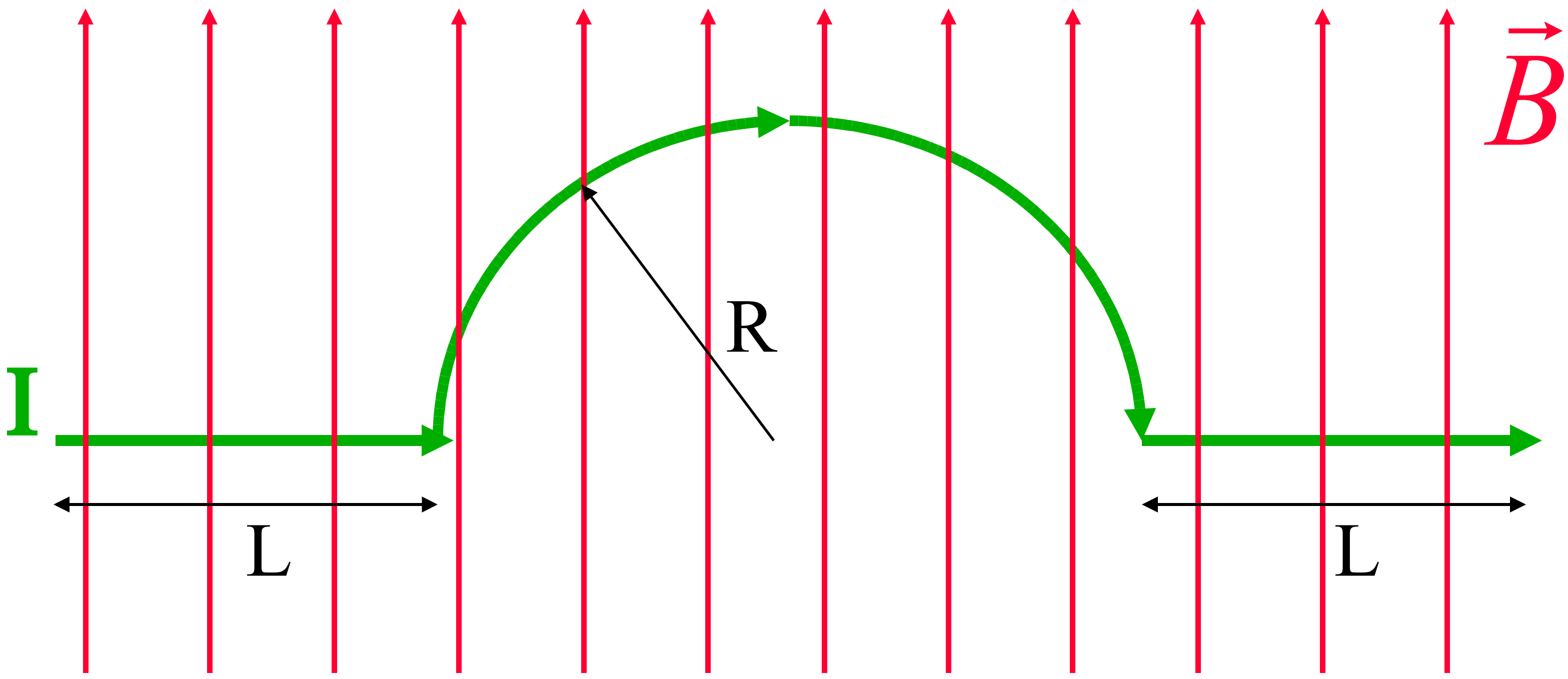
To find the force exerted by a non-uniform magnetic field on a curved current we divide the conductor in small sections dL and add (integrate) the forces exerted on every section dL.

Then, for the total length of the curved conductor in a non-uniform magnetic field

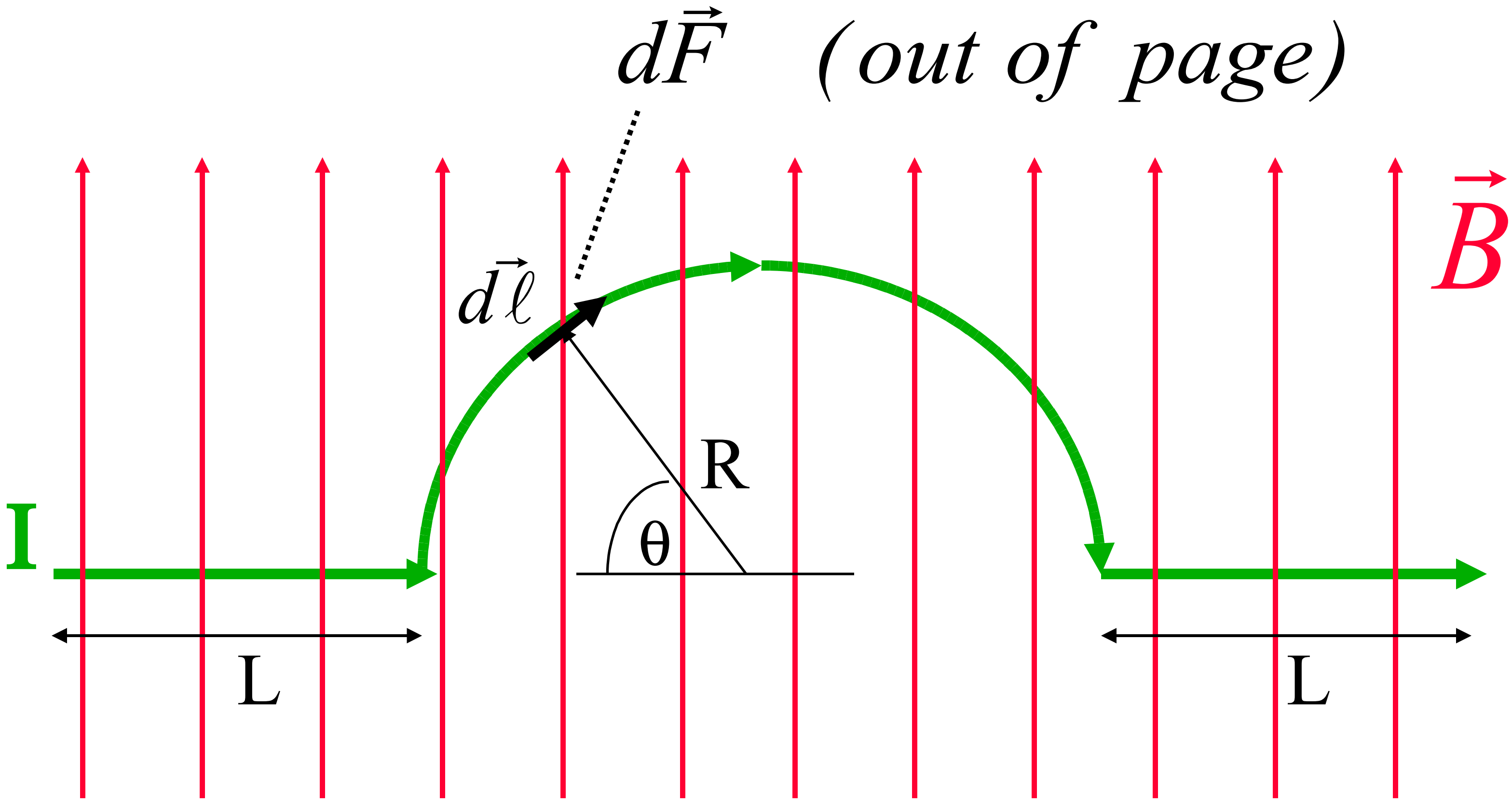
$$\underline{F} = \int \underline{dF} = \int \underline{dL}\ I \times \underline{B}$$

Nonuniform Fields and Curved Conductors: Example

- What is the force on the current-carrying conductor shown?

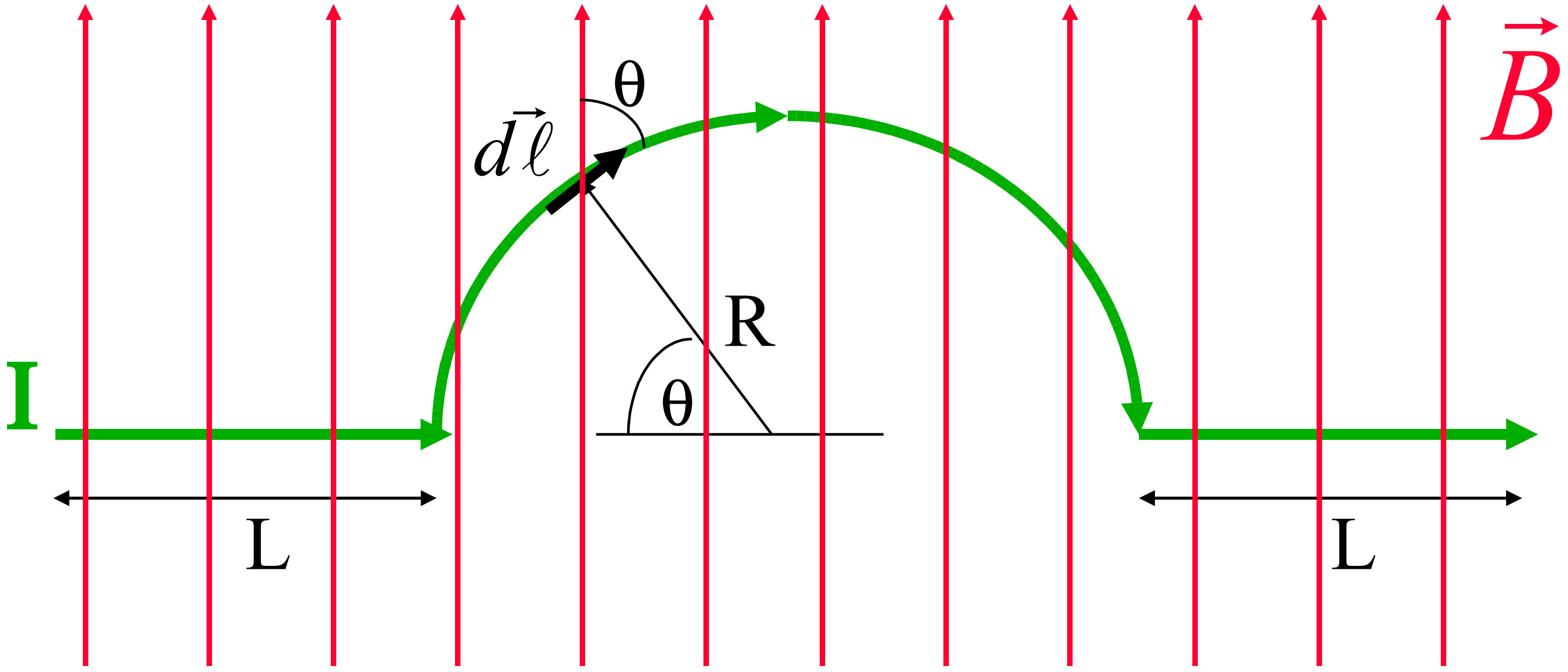


Nonuniform Fields and Curved Conductors: Example



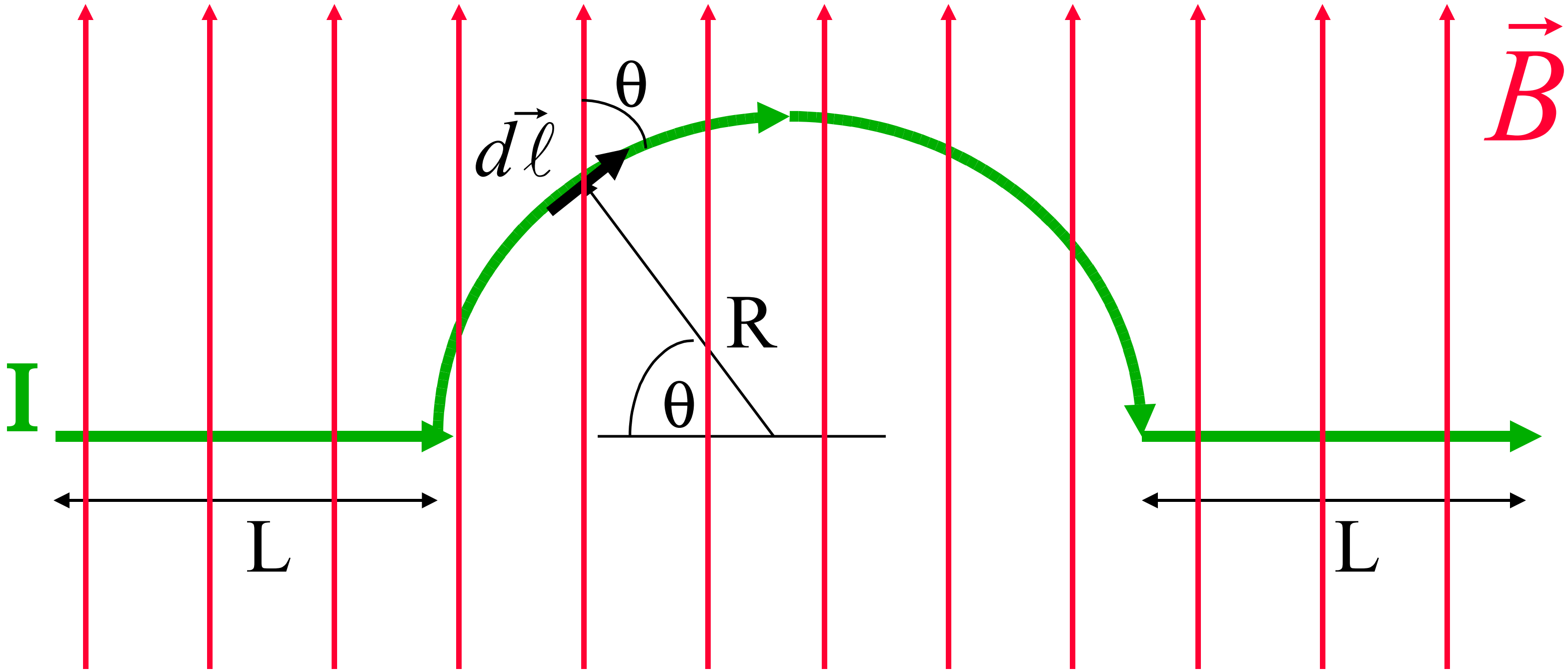
Nonuniform Fields and Curved Conductors: Example

$$\left|d\vec{F}\right| = \left|I d\vec{\ell} \times \vec{B}\right| = I\,d\ell\,B\sin\theta$$



Nonuniform Fields and Curved Conductors: Example

$$\left|d\vec{F}\right| = \left|I\,d\vec{\ell} \times \vec{B}\right| = I\,d\ell\,B\sin\epsilon \qquad d\ell = R\,d\theta$$



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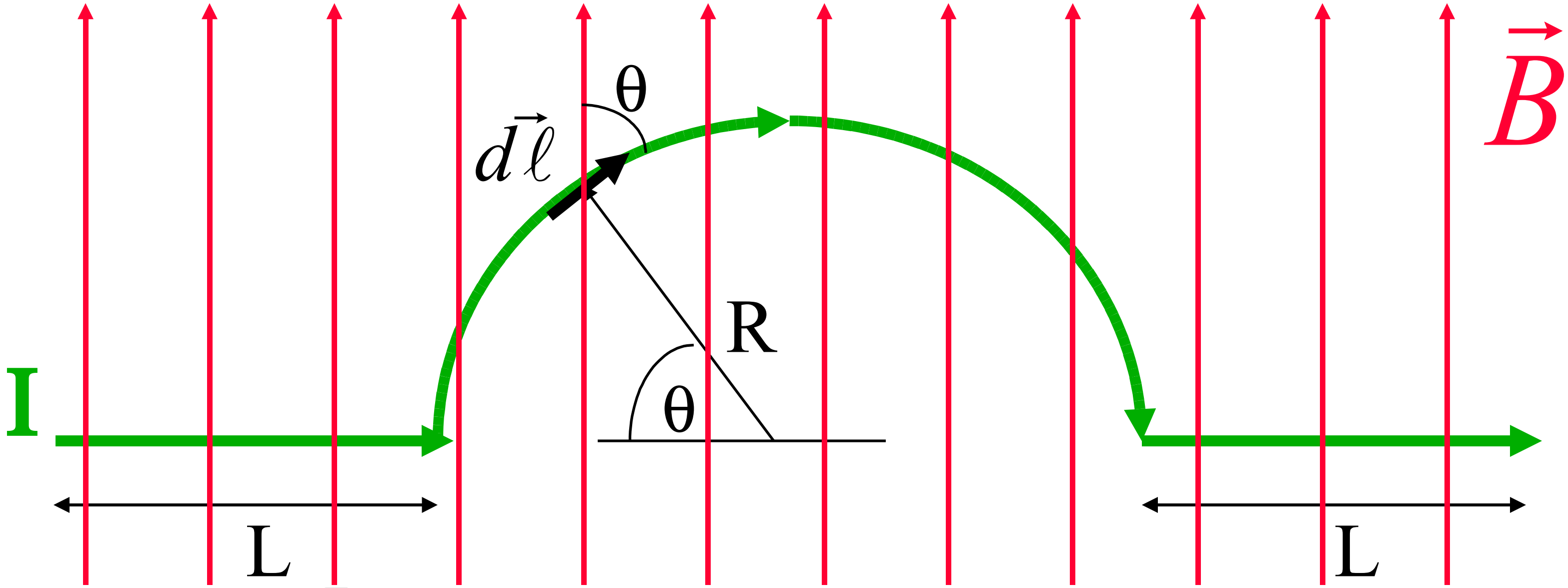

$$F = IBR\int_0^{\pi} \sin\theta \; d\theta$$

Nonuniform Fields and Curved Conductors: Example

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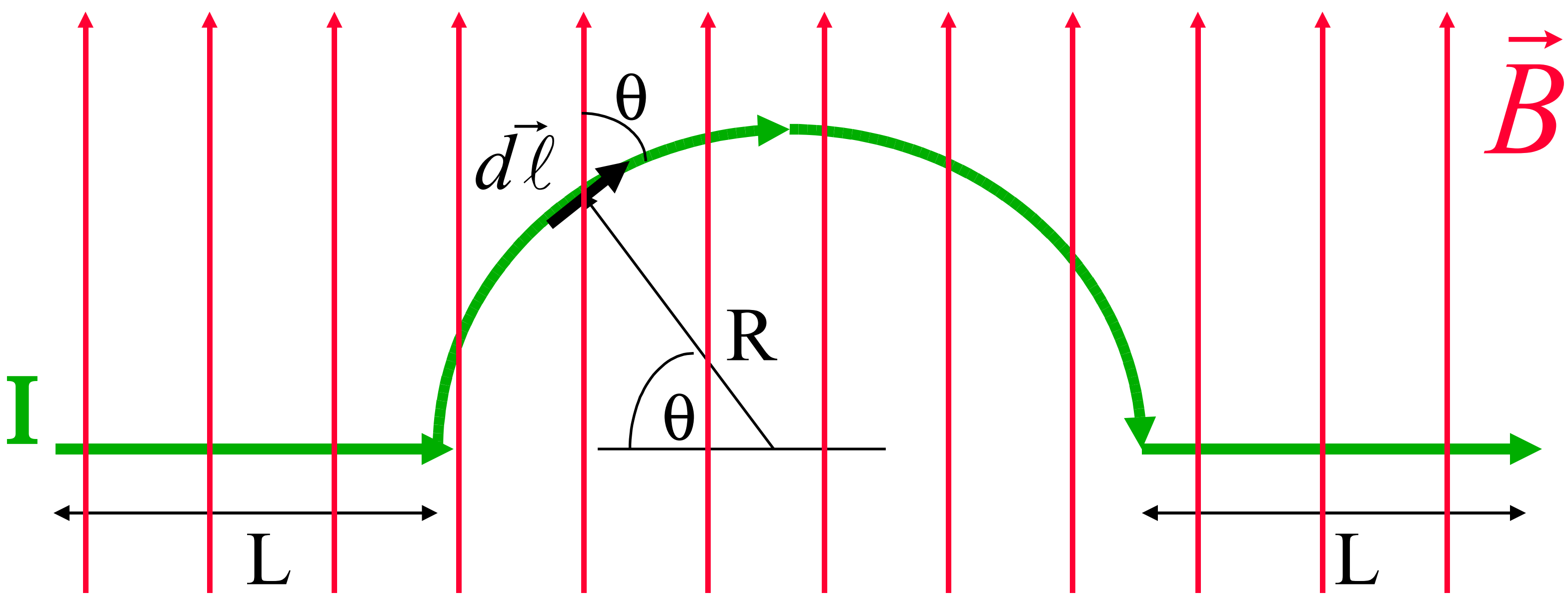

$$F = IBR\int_0^{\pi} \sin\theta\; d\theta = -IBR\left|\cos\theta\right|_0^{\pi}$$

Nonuniform Fields and Curved Conductors: Example

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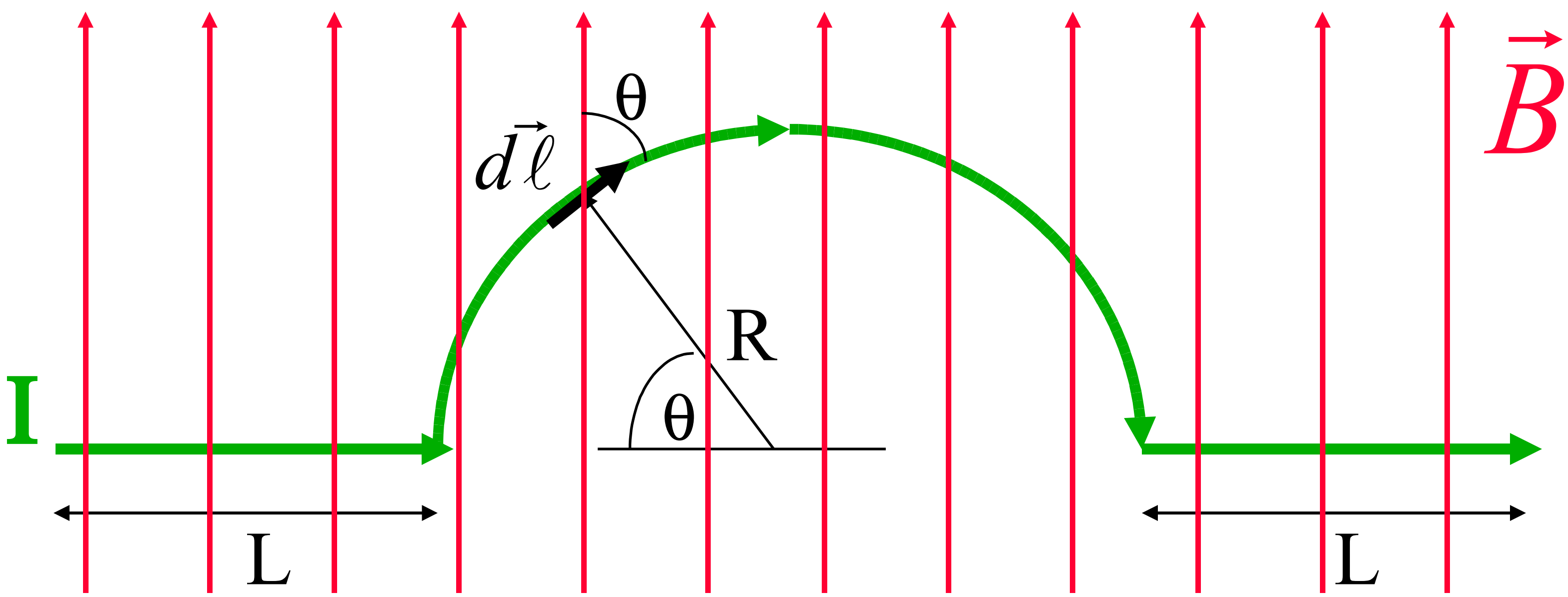
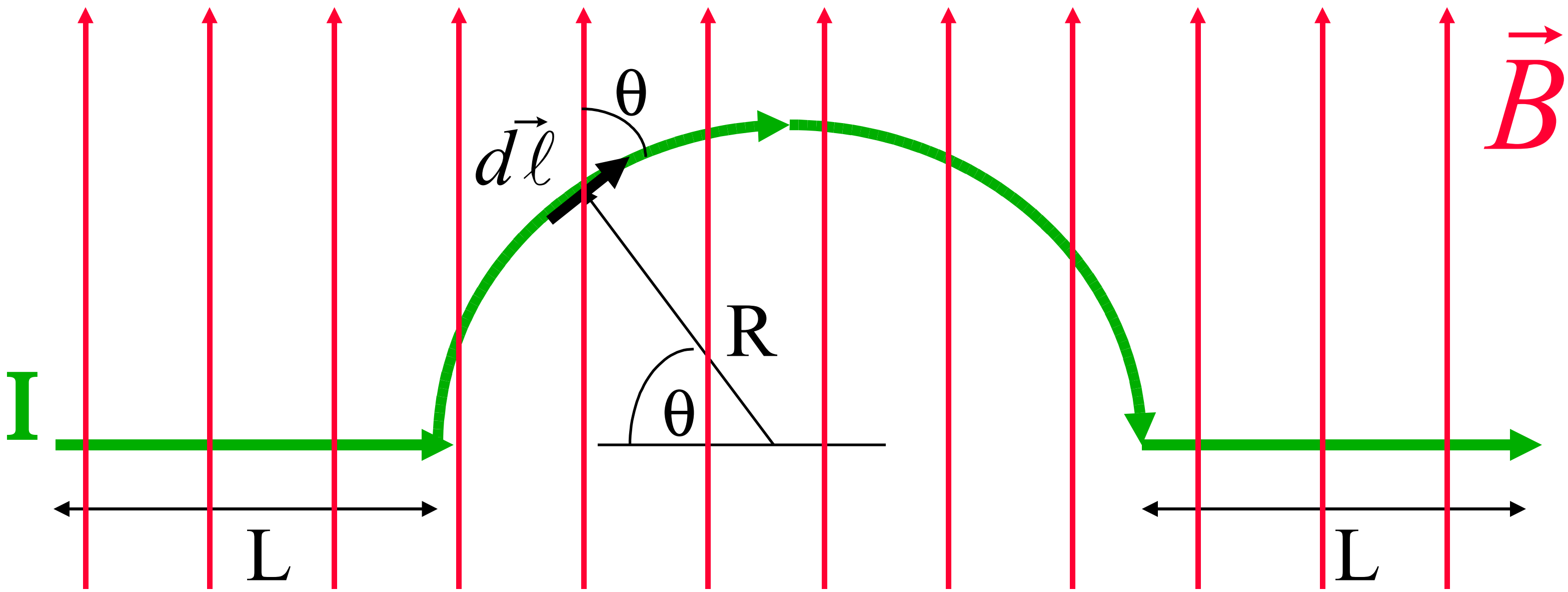


$$F = IBR\int_0^{\pi} \sin\theta\; d\theta = -IBR\left|\cos\theta\right|_0^{\pi} = 2IBR$$

Nonuniform Fields and Curved Conductors: Example

$$\therefore\ F_{tot} = 2IBR + 2IBL$$



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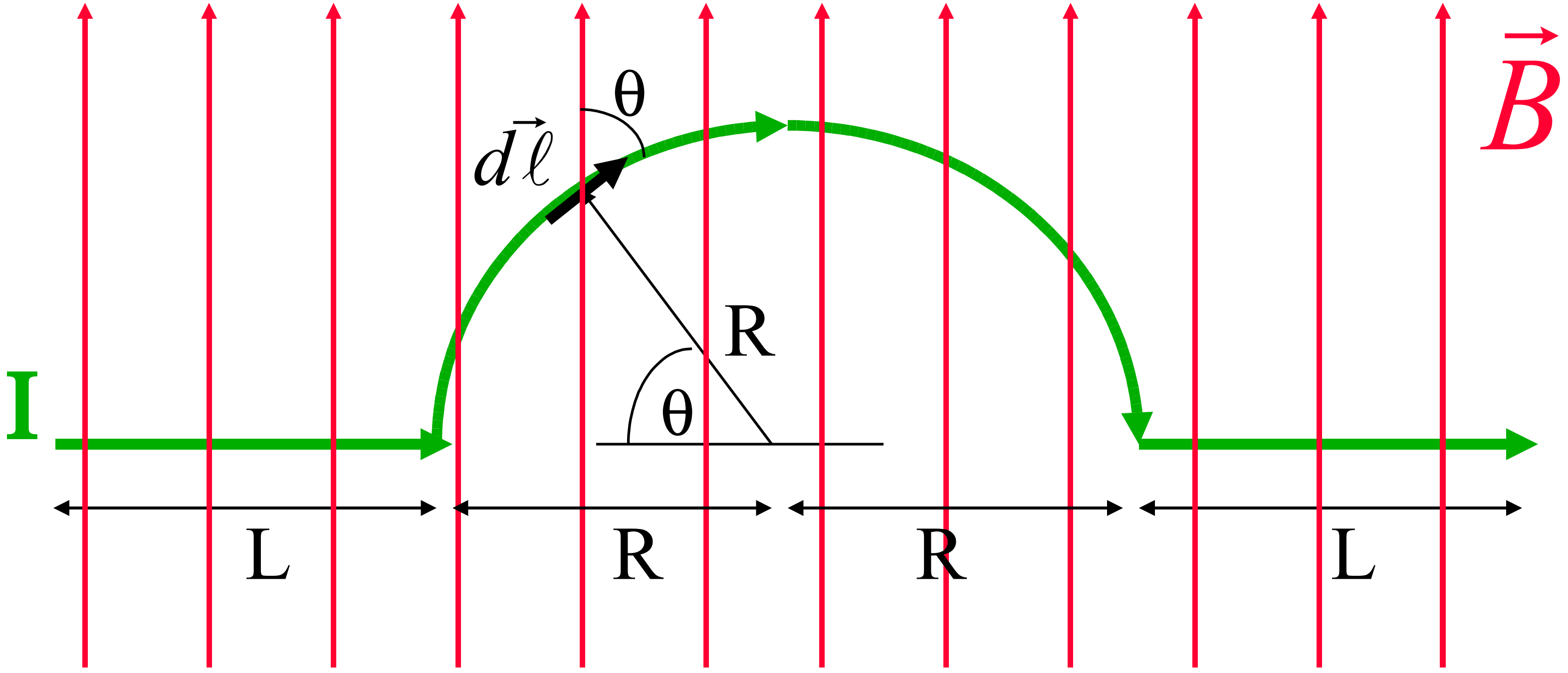
Nonuniform Fields and Curved Conductors: Example

$$\therefore\ F_{tot} = 2IBR + 2IBL = IB(2L + 2R)$$



Equal to the force we would find for a straight wire of length 2(R+L)